LA-UR -81-2505

CONF-8106156--1

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TITLE THE NEUTRINO NUMBER OF THE UNIVERSE

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SUBMITTED TO To be published in the proceedings of the 1981 International Conference on Neutrino Physics and Astrophysics, Maui, Hawaii, June 30-July 9, 1981.

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August 10, 1981

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THE NEUTRINO NUMBER OF THE UNIVERSE*

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Abstract

The influence of grand unified theories on the lepton number of the universe is reviewed. A scenario is presented for the generation of a large (>> 1) lepton number and a small (<< 1) baryon number.

Within the past two years it has been realized that if current ideas about Grand Unified Theories (GUTs) are correct, they may provide the answer to a fundamental cosmological problem: the origin of the baryon asymmetry.[1] It seems natural to speculate that the observed preponderance of baryons over antibaryons is the result of the baryon number (B), charge conjugation (C), and charge conjugation-parity (CP) violating interactions of the super-massive ($m \gtrsim 10^{14}$ GeV) bosons that are intrinsic to GUTs. In this talk I would like to discuss the implications of GUTs for the lepton number (L) of the universe.

The overall charge neutrality of the universe requires that the excess of protons over antiprotons be balanced by a corresponding excess of electrons over positrons:

*Talk presented at the 1981 International Conference on Neutrino Physics and Astrophysics.

†Work supported in part by the Department of Energy.

$$L_e = \frac{n_e - n_{\bar{e}}}{n_\gamma} = \frac{n_p - n_{\bar{p}}}{n_\gamma} \cong B \cong 10^{-9}, \tag{1}$$

where n_i is the present number density of species i. Therefore any large [> 0(1)] lepton number in the universe must be due to an excess of neutrinos over antineutrinos.

The best limit on the neutrino number of the universe comes from the limit on the total energy density of the universe. In the absence of a large cosmological constant, the present total energy density, ρ_o, may be expressed in terms of the Hubble constant H_o, the deceleration parameter q_o, and the Planck mass $m_p = 1.2 \times 10^{19}$ GeV, as[2]

$$\rho_o = 2q_o \left(\frac{3H_o^2 m_p^2}{8\pi}\right). \tag{2}$$

The observational limits,[3] $100 > H_o$ (km $s^{-1}Mpc^{-1}$) > 50, and $|q_o| < 2$, require the present energy density of the universe to be $\rho_o < 8 \times 10^{-29}$ g cm^{-3}. This limits the contribution of primordial neutrinos to the energy density, and hence limits the neutrino number of the universe. If we assume that neutrinos were relativistic when they decoupled in the early universe (i.e., $m_\nu \ll 1$ MeV) and that they have only one spin state, then the contribution of the primordial neutrinos to the present energy density would be

$$\rho_\nu(T_o) = -\left(\frac{T_o}{T_D}\right)^4 \frac{3T_D^4}{2\pi^2} \; Li_4(-e^{\mu_D/T_D}), \tag{3}$$

where μ_D and T_D are the neutrino chemical potential and neutrino temperature of decoupling, T_o is the present neutrino temperature, and Li_n is the polylogarithm function.[4] Equation (3) has the "hot" ($\mu/T \ll 1$) and "cold" ($\mu/T \gg 1$) expansions[4]

. (8)

ıc universe limits the neutrino number

$$\rho_\nu(T_o) = \left(\frac{T_o}{T_D}\right)^4 \frac{21}{8\pi^2} \zeta(3)T_D^4\left[1 + \frac{\mu}{T}\right.$$

[redacted]

$$= \left(\frac{T_o}{T_D}\right)^4 \frac{\mu_D^4}{8\pi^2}\left[1 + 12\zeta(2)\left(\frac{T}{\mu}\right.\right.$$

In the standard cosmological model (μ

trino energy density in the cold limit

$$\rho_\nu(T_o) = \frac{\mu_o^4}{8\pi^2}\left[1 + 12\zeta(2)\left(\frac{T_o}{\mu_o}\right)^2 + \right.$$

$$\left. \frac{D}{D}\frac{6}{7}\frac{\zeta(3)}{\zeta(4)} + \ldots\right] \quad (\mu/T \ll 1)$$

where μ_o is the present neutrino ch

limit of Equation (2) then implies

$$\left.\frac{D}{D}\right)^2 + \ldots\Bigg] \quad (\mu/T \gg 1) \ . \qquad (4)$$

$$\mu_o \lesssim 1.3 \times 10^{-2} \text{ eV}.$$

The present neutrino number density is $_D/T_D) = (\mu_o/T_o)$, and the present neu-

is

$$n_\nu(T_o) = -\left(\frac{T_o}{T_D}\right)^3\left[\frac{T_D^3}{\pi^2} Li_3(-e^{-\mu_D/T_D})\right]$$

$$\ldots\Big] \quad (\mu_o/T_o \gg 1) \ , \qquad (5)$$

which has the cold expansion

emical potential. The observational

$$n_\nu(T_o) = \frac{\mu_o^3}{6\pi^2}\left(1 + 6\zeta(2)\left(\frac{T_o}{\mu_o}\right)^2 + \ldots\right.$$

(6)

Therefore the total energy density of th

to be given by

, (7)

$$\left|\frac{n_\nu}{n_\gamma}\right| \le 8 \times 10^4 \ , \tag{9}$$

where for n_γ we have used $n_\gamma = 400\ \mathrm{cm}^3$. Therefore the only reliable limit allows the lepton number of the universe to be large.

The existence of a large neutrino degeneracy would have several interesting cosmological effects. In particular, it would largely determine the results of primordial nucleosynthesis.[5] The primordial ^{4}He abundance is particularly sensitive to the value of the neutrino chemical potential. The existence of a large neutrino degeneracy may also prevent the high-temperature restoration of spontaneously broken gauge symmetries and the associated phase transitions.[6] This would prevent the possibility of any exponential expansion[7] and dissolve bounds on Higgs masses found by limiting the entropy produced in phase transitions.[8] It would also solve the problem of excess heavy stable monopoles produced in the phase transitions of hot models.[9] A large neutrino chemical potential may also make present-day detection of the background neutrinos possible due to the increase in number and energy of the neutrinos over the case of zero chemical potential.

The current folklore maintains that Grand Unified Theories would take any large asymmetry and make a lepton number comparable to the baryon number, hence small.[10] However we shall see below that this need not be the case, that Grand Unified Theories (GUTs) need not eradicate the memory of initial conditions. In order to illustrate the possibility of a large lepton number, we will consider two models; a model based on SU(5), and a model based on SO(10).

An SU(5) family[11] of fermions consisting of fifteen left-handed fermion fields is placed into the reducible representation $\bar{5}_f \oplus 10_f$. Such a family has the generic particle content

$$\bar{5}_f = (\vec{D}^C_L,\ \nu_L,\ E_L)$$

$$10_f = (\vec{U}_L,\ \vec{U}^C_L,\ \vec{D}_L,\ E^C_L) \tag{10}$$

where U, D, ν, and E represent the charge 2/3 quark, the charge -1/3 quark, the neutrino, and the charged lepton in the family. The subscript L indicates projection of the left-handed component and the superscript C indicates the charge conjugate state.

The vector bosons transform as the adjoint 24-dimensional representation and have gauge couplings to the fermions

$$L_g = \frac{g}{\sqrt{2}} [\bar{5}_f \cdot 5_f + \overline{10}_f \cdot 10_f] \, 24_V \tag{11}$$

where g is the gauge coupling constant.

The Higgs bosons with Yukawa couplings are usually taken to be in the 5-dimensional representation with coupling to fermions

$$L_Y = [10_i (h_U)^{ij} 10_j] \cdot 5_H + [\bar{5}_i (h_D)^{ij} 10_j] \cdot \bar{5}_H, \tag{12}$$

where i and j are family indices and h_U and h_D are the Yukawa coupling matrices.

If we consider only vector couplings, then it is clear that the couplings in Eq. (11) are invariant under two global phase transformations. The corresponding conserved quantum numbers are given by $\chi_5 = +1(-1)$ for each field in the $5_f(\bar{5}_f)$ and $\chi_{10} = +1(-1)$ for each field in the $10_f(\overline{10}_f)$. Scalar interactions violate χ_5 and χ_{10}, but from Eq. (11) we see that it is possible to take a linear combination of χ_5 and χ_{10} that is still a conserved quantum number, $Z = 3(-3)$ for $5_f(\bar{5}_f)$, $Z = +1(-1)$ for $10_f(\overline{10}_f)$, and $Z = -2(+2)$ for the $5_H(\bar{5}_H)$. When $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ breaks to $SU(3)_C \otimes U(1)_{EM}$, Z is spontaneously broken, but a combination of Z and the hypercharge remains unbroken. This combination is just the baryon number minus the lepton number, B-L. Although the full SU(5) theory does not separately conserve χ_5 and χ_{10}, analytic and numerical results indicate that to a good approximation if the scalar mass is sufficiently large, scalar interactions may be neglected.[4,12] In this approximation χ_5 and χ_{10} are separately conserved.[13]

The vector interactions, however, are faster than the Higgs interactions and may even be infinite.[4] The effect of the vector interactions will be to distribute any asymmetry in fermion fields equally among all members of the irreducible representation containing the fermion (assuming that the charges associated with all gauged quantum numbers are zero). Therefore, as initial conditions we need only consider "Gauge Invariant Initial Conditions" in which all members of a given irreducible representation have equal asymmetries. Therefore any initial asymmetry in fermion fields may be represented by two numbers, χ_5 and χ_{10}.

From Eq. (10) it is obvious that with gauge invariant initial conditions the baryon and lepton numbers are given by

$$B \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = \eta_5 + \eta_{10}$$

$$L \equiv \frac{n_\ell - n_{\bar{\ell}}}{n_\gamma} = 3\eta_5 + 2\eta_{10} \ , \tag{13}$$

where η_i is the asymmetry in the fermion fields of the ith representation. Since χ_5 and χ_{10} are conserved by vector interactions, and the Higgs interactions may be ignored (at least until after the baryon synthesis era) the fact that B and L are linearly independent means that it is possible to have a large L and a small B. The conditions for this are that $\eta_5 + \eta_{10} = 0$, but η_5 and η_{10} must both be large. A cancellation of two large numbers seems unnatural within the context of SU(5), but it has a natural explanation if SU(5) is embedded in an SO(10) gauge theory.

In grand unified theories based on the gauge group SO(10),[11] all the fermions in a single family are assigned to the complex spinor representation, 16_f:

$$16_f^T = (\vec{U}_L,\ \vec{U}_L^C,\ \vec{D}_L,\ \vec{D}_L^C,\ E_L,\ E_L^C,\ \nu_L,\ N_L^C) \ . \tag{14}$$

Since there are only 15 known fermion fields per family (assuming the existence of the top quark) it is necessary to postulate the existence of a particle, the N_L^C, that is a singlet under $SU(3)_C \otimes SU(2)_L \otimes U(1)$. The existence of this particle has interesting consequences for the lepton number of the universe as well as for low energy neutrino phenomenology.

The gauge vector bosons in SO(10) transform as the 45-dimensional adjoint representation. The gauge coupling to fermions has the form

$$L_g = \frac{g}{\sqrt{2}} \overline{16}_f \cdot 16_f \cdot 45_V . \tag{15}$$

The vector interactions in SO(10) conserve a quantum number $\chi_{16} = +1(-1)$ for each field in the $16_f(\overline{16}_f)$, analogous to the χ_5 and χ_{10} conservation in SU(5).

The Higgs fields which can couple to fermions appear in the decomposition of $16 \otimes 16$:

$$16 \otimes 16 = (10 + 126)_S + (120)_A . \tag{16}$$

If the N_L^C acquires a very large Majorana mass M_N presumably through a non-zero vacuum expectation value for the 126_H or through radiative corrections, then the neutral lepton mass matrix in the ν, N basis will have the form[14]

$$\begin{pmatrix} 0 & m_q \\ m_q & M_N \end{pmatrix}$$

where m_q is the mass of the charge 2/3 quark in the family. The approximate eigenvalues of this matrix are m_q^2/M_N and M_N. The observed low-energy neutrinos will thus have masses $0(m_U^2/M_N)$ which can be made compatible with present observations if M_N is sufficiently large.

We now consider the damping of asymmetries in SO(10) models with an initial asymmetry η_{16}^{o} in each member of the 16_f:

$$\vec{U}_- = \vec{U}_-^C = \vec{D}_- = \vec{D}_-^C = E_- = E_-^C = \nu_- = N_-^C = \eta_{16}^0 \quad .$$

It is obvious that in the limit of exact SO(10) invariance the presence of an unbroken charge conjugation operator requires all asymmetries in quantum numbers that are odd under C (e.g., B, L, Q, ...) to vanish. Consider a universe with a large initial fermion asymmetry. The Higgs interactions will slowly bleed the initial asymmetries. In the limit of exact SO(10) invariance the C symmetry will bleed all the fermion fields at an equal rate keeping B and L zero. However, once SO(10) breaks due to a large Majorana mass for the N_L^C there will be a large disparity in the rate that the asymmetry in the N_L^C is driven to zero and the rate that the asymmetries in the rest of the fermion fields are driven to zero. As the asymmetry in N_L^C is driven to zero, the vector interactions will redistribute the asymmetry in the other fields. Since there are vector bosons that connect the N_L^C with the quarks, such a redistribution may generate a baryon number.[15] The magnitude of B depends on the amount of enhanced N_L^C depletion relative to the depletion in the light fermion fields when the lightest vector boson connecting the N_L^C to light fermions decouples at temperatures less than the vector mass m_{v1}. B can be large or small depending on the mass of the N_L^C. A large N_L^C mass results in large relative N_L^C depletion at $T = m_{v1}$ and severe rearrangement of the initially C-invariant asymmetries, hence a potentially large B. A small N_L^C mass results in a small N_L^C relative depletion at m_{v1} which results in a small B since the original C-symmetry remains largely intact.

So far a lepton number is generated with L = B. However, once $T \lesssim m_{v1}$ the baryon number will be frozen in while the lepton number will continue to grow. In particular as the asymmetry in N_L^C continues to be driven to zero, the C symmetry is badly broken because the large asymmetry in the ν_L is now uncancelled since there is no asymmetry in the N_L^C. (In fact for $T \lesssim M_N$, the assignment of a lepton number to N_L^C becomes meaningless.) Therefore a large lepton number is likely to result.

In the scenario outlined here, a universe with a large gauge invariant initial asymmetry, but zero baryon and lepton numbers may evolve into a universe with $L \gg B$.

In conclusion, it was demonstrated that the interactions present in Grand Unified Theories combined with CP non-invariant initial fermion asymmetries can naturally lead to the present lepton number of the universe being much larger than the baryon number. This is in disagreement with others who have claimed that $L \sim B$ as a consequence of grand unification. In SU(5) models the requirement that $L \gg B$ requires a cancellation between different contributions to the initial baryon number. This cancellation has a natural explanation in SO(10) models where all fermions in a family are placed in a single irreducible representation. While we have considered explicitly only SU(5) and SO(10) unified models, the results can be easily generalized to other theories. The reason that SU(5) and SO(10) theories allow $L \gg B$ can be related to the fact that they also predict a discrepancy between quark and neutrino masses. In SU(5), $m_\nu = 0$ as a result of the global B-L symmetry which in turn is related to the reducibility of the fermion representation. It is this reducibility that allows B and L to be independent. In SO(10), $m_\nu \ll m_q$ is a result of a large $SU(3)_C \times SU(2)_L \times U(1)$ invariant Majorana mass term for the right-handed component of the neutrino. This mass term rapidly destroys any net lepton number residing in the right-handed neutrino field thus leaving an initial asymmetry in the left-handed neutrino unbalanced. We expect that any theory that predicts $m_\nu \ll m_q$ in a natural way will also allow $L \gg B$.

A more thorough discussion of the lepton number in GUTs may be found in Ref. 15.

I would like to thank Jeff Harvey, whose collaborative efforts led to the work reported here, and Stephen Wolfram, with whom some of the seminal work was done.

REFERENCES

1. For a thorough discussion of baryon number generation in grand unified models and a list of original references, see J. Harvey, E. Kolb, D. Reiss, and S. Wolfram, "Calculation of Cosmological Baryon Asymmetry in Grand Unified Gauge Models," Caltech preprint CALT-68-850.
2. For a review of the standard big bang, see e.g., S. Weinberg, Gravitation, (Wiley, New York, 1972) Chapter 15.
3. J. Kristian, A. Sandage, and J. Westphal, Astrophys. J. 221, 383 (1978).
4. E. Kolb and S. Wolfram, Nucl. Phys. B 172, 223 (1980).
5. R. V. Wagoner, W. A. Fowler, and F. Hoyle, Astrophys. J. 148, 3 (1967); A. Yahil and G. Baudet, Ap. J. 206, 261 (1976); Y. David and H. Reeves, in Proceedings of Les Houches Summer School, vol. XXXII, R. Balian, J. Audouze, and D. Schramm, eds. (North-Holland, New York, 1980).
6. A. D. Linde, Phys. Rev. D14, 3345 (1976).
7. E. W. Kolb and S. Wolfram, Astrophys. J. 239, 428 (1980).
8. A. H. Guth and E. J. Weinberg, Phys. Rev. Lett. 45, 1131 (1980); E. Witten, Nucl. Phys. B177, 477 (1981); M. A. Sher, Phys. Rev. D22, 1989 (1980).
9. J. Preskill, Phys. Rev. Lett. 43, 1365 (1979).
10. S. Dimopoulos and G. Feinberg, Phys. Rev. D20, 1283 (1979); D. Schramm and G. Steigman, Phys. Lett. 87B, 14 (1979); D. V. Nanopoulos, D. Sutherland, and A. Yildiz, Lett. Nuovo Cim. 28, 205 (1980); M. S. Turner, Phys. Lett. 98B, 145 (1981).
11. For a review of GUTs, see e.g., P. Langacker, "Grand Unified Theories and Proton Decay," SLAC preprint SLAC-PUB-2544.
12. J. Fry, K. Olive, and M. Turner, Phys. Rev. D22, 2953 (1980).
13. S. Treiman and F. Wilczek, Phys. Lett. 95B, 222 (1980).
14. M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity (North-Holland, New York, 1979) ed. P. van Niewenhuizen and D. Freedman.
15. J. Harvey and E. Kolb, Phys. Rev. D, to be published.